Close limit evolution of Kerr-Schild type initial data for binary black holes

Olivier Sarbach
Center for Gravitational Physics and Geometry, Department of Physics,
The Pennsylvania State University, 104 Davey Lab, University Park, PA 16802.

Manuel Tiglio
Center for Gravitational Physics and Geometry, Department of Physics and Department of Astronomy and Astrophysics,
The Pennsylvania State University, 525 Davey Lab, University Park, PA 16802.

Jorge Pullin
Department of Physics and Astronomy, Louisiana State University, 202 Nicholson Hall, Baton Rouge, LA 70803-4001.

We evolve the binary black hole initial data family proposed by Bishop *et al.* in the limit in which the black holes are close to each other. We present an exact solution of the linearized initial value problem based on their proposal and make use of a recently introduced generalized formalism for studying perturbations of Schwarzschild black holes in arbitrary coordinates to perform the evolution. We clarify the meaning of the free parameters of the initial data family through the results for the radiated energy and waveforms from the black hole collision.

I. INTRODUCTION

Collisions of binary black holes are expected to be one of the primary sources of gravitational radiation to be detected by interferometric gravitational wave detectors. Given the non-symmetric, time-dependent nature of the problem, the only realistic hope of modeling a collision is via numerical simulations. Unfortunately, given computer limitations, it is expected that in the near future most evolutions will have to start with the black holes quite close to each other. This brings to the forefront the problem of specifying initial data for the binary black hole collisions. Ideally, one would like to have initial data representing "astrophysically relevant" situations, that is, resembling the situation the two black holes would be in when they are in a realistic collision at the given separation. Unfortunately, providing realistic data is tantamount to solving the evolution problem. Since this cannot be done, one is left with generating families of initial data based on mathematical or computational convenience. Initial attempts to provide initial data concentrated on solutions to the initial value problem that were conformally flat [1]. Conformally flat spatial metrics simplify considerably the constraint equations but suffer from drawbacks, most notably the inability to incorporate Kerr black holes, which are not known to have conformally flat sections (for a perturbative proof of non-existence, see [2]). Recently, attention has been drawn to the construction of initial data based on the Kerr–Schild form of the Schwarzschild and Kerr metrics. These constructions have several attractive properties: the slices are horizon-penetrating, which makes them suitable for the application of the "excision" technique for evolving black holes, and they can naturally incorporate boosted and spinning black holes [3].

There have been two different proposals to use Kerr–Schild coordinates for binary black holes. In the proposal of Huq, Matzner and Shoemaker [4], two black holes individually in Kerr–Schild form were superposed. In the proposal of Bishop *et al.* [5] the superposition was carried out in a way that the resulting superposed metric was in Kerr–Schild form. This latter proposal has the property that in the "close limit" in which the separation of the holes is small, the metric is given by a distorted Kerr-Schild black hole.

In this paper we will consider the evolution of the Bishop *et al.* family of initial data in the close limit, by treating the space-time as a small perturbation of a non-rotating Kerr–Schild black hole. The evolution will be carried out using a recently introduced perturbative formalism that allows to evolve Schwarzschild black holes in arbitrary spherically symmetric coordinates [6]. We will present an explicit solution to the initial value problem posed by Bishop *et al.* in the close limit and use it to compute the radiated energy in the black hole collision. This in turn will help use clarify the meaning of free parameters that appear in these families of data.

II. KERR-SCHILD INITIAL DATA

In their paper, Bishop *et al.* [5] assume that at the initial slice, the three metric and the extrinsic curvature are of Kerr-Schild (KS) type. The KS space-time metric is defined by

$$g_{\mu\nu} = \eta_{\mu\nu} - 2V k_\mu k_\nu \, ,$$

where k_μ is null. The "background" metric $\eta_{\mu\nu}$ is taken to be the Minkowski metric with coordinates $(t, \underline{x}) = (t, x^i)$ such that $\eta_{tt} = -1$, $\eta_{ti} = 0$ and $\eta_{ij} = \delta_{ij}$. The null vector k_μ satisfies $k_t = -1$ and $k^i k_i = 1$, where $k^i = \delta^{ij} k_j$. As an example, for the Schwarzschild geometry,

$$V = -\frac{M}{r}\,, \quad k_i dx^i = -dr.$$

For a KS metric, the three metric and extrinsic curvature with respect to a slice $t = \text{const.}$ are

$$\bar{g}_{ij} = \delta_{ij} - 2V k_i k_j, \tag{1}$$

$$K_{ij} = -\frac{1}{\alpha}\partial_t \left(V k_i k_j\right) + 2\alpha \left[V k^s \nabla_s \left(V k_i k_j\right) - \nabla_{(i}\left(V k_{j)}\right)\right], \tag{2}$$

where $\alpha = (1 - 2V)^{-1/2}$ is the lapse and where ∇ refers to the flat metric δ_{ij}. Bishop *et al.*'s solution procedure consists in inserting (1,2) into the constraint equations and to solve the resulting equations for V, $\dot{V} = \partial_t V$ and $\dot{k}_i = \partial_t k_i$, where k_i is assumed to be given. (k_i has only two independent components since $k^i k_i = 1$.) Below, we review the discussion of these equation where a particular ansatz is made for k_i representing two nearby non-rotating and non-spinning black holes.

A. Two black hole data

A single Schwarzschild black hole can be represented as

$$k_i = \frac{\nabla_i \phi}{|\nabla \phi|}\,, \quad |\nabla\phi|^2 = \delta^{ij} \nabla_i \phi \cdot \nabla_j \phi,$$

with $\phi = 1/r$. For two black holes, Bishop *et al.* make the ansatz

$$\phi(\underline{x}) = \frac{M_1}{|\underline{x} - \underline{x}_1|} + \frac{M_2}{|\underline{x} - \underline{x}_2|}\,,$$

where $\underline{x}_{1,2}$ denote the position of the black hole $1,2$ which has mass $M_{1,2}$. If the black holes are located at $\underline{x}_i = a_i(0,0,1)$, with $a_1 > 0 > a_2$, ϕ may be expanded in a sum over multipoles:

$$\phi(r, \vartheta) = \sum_{\ell=0}^{\infty} \frac{M_1 a_1^\ell + M_2 a_2^\ell}{r^{\ell+1}} P_\ell(\cos\vartheta), \tag{3}$$

where P_ℓ denote standard Legendre polynomials and where (r, ϑ, φ) are polar coordinates for $\underline{x}$. It is important to note that the expansion (3) is only valid for $r > \max\{a_1, -a_2\}$.

Defining the separation parameter

$$\varepsilon = \frac{a_1 - a_2}{M}\,,$$

where $M = M_1 + M_2$ is the total mass, and imposing the center of mass condition $M_1 a_1 + M_2 a_2 = 0$, the close limit of (3) becomes

$$\phi = \frac{M}{r} + \varepsilon^2 \frac{M M_1 M_2}{r^3} P_2(\cos\vartheta) + \mathcal{O}(\varepsilon^3/r^4).$$

As a result, to first order in ε^2, k_i is given by

$$k_r = 1, \quad k_A = \varepsilon^2 \frac{M_1 M_2}{r} \hat{\nabla}_A P_2\,, \tag{4}$$

where here and in the following, $A = \vartheta, \varphi$. The remaining amplitudes are expanded according to

$$V = -\frac{M}{r} + \varepsilon^2 v(r) P_2\,, \quad \dot{V} = \varepsilon^2 \dot{v}(r) P_2\,, \quad \dot{k}_A = \varepsilon^2 \dot{k}(r) \hat{\nabla}_A P_2\,. \tag{5}$$

(In Bishop *et al.*'s notation: $\dot{v} = v_T$ and $\dot{k} = -k_T$.) Introducing this into the constraint equations, and keeping only terms of the order ε^2, one obtains the equations

$$0 = -\dot{v} + \frac{3M}{r^2}\left(1 + \frac{2M}{r}\right)\dot{k} - \frac{3}{r}v - \frac{6MM_1M_2}{r^5}(r - M), \tag{6}$$

$$0 = -v' - \frac{4}{r}v + \frac{6M^2}{r^3}\dot{k} - \frac{6MM_1M_2}{r^5}(r - M), \tag{7}$$

$$0 = -M\dot{k}' + v + \frac{2M}{r}\dot{k} + \frac{6MM_1M_2}{r^3}. \tag{8}$$

Here, a prime denotes differentiation with respect to r. The system (7,8) can be re-expressed as a single second order equation. Introducing the dimensionless quantities $x = r/M$ and $\mu = M_1M_2/M^2$, this equation reads

$$0 = -v_{xx} - \frac{5}{x}v_x + \frac{6}{x^3}v + \frac{6\mu}{x^6}(3x + 2). \tag{9}$$

Once we have solved this equation, the remaining amplitudes $\dot{k}$ and $\dot{v}$ are obtained from (6) and (7), respectively.

B. The solutions of equation (9)

A particular solution of (9) is given by

$$v(x) = -\frac{2\mu}{3}\left(1 - \frac{2}{x} + \frac{3}{x^2} + \frac{3}{x^3}\right).$$

In order to find the solutions of the homogeneous equation, one performs the transformations $x = 24/z^2$, $v(x) = z^4 u(z)$, which yields the Bessel differential equation

$$0 = z^2 u_{zz} + z u_z - (16 + z^2)u.$$

The solutions are a linear combination of the Bessel functions $J_4(iz)$ and $Y_4(iz)$. While $J_4(iz)$ behaves as z^4 for small $|z|$, $Y_4(iz)$ has the expansion [7]

$$\begin{aligned} Y_4(iz) &= -\frac{96}{\pi}\, z^{-4}\left(1 - \frac{z^2}{12} + \frac{z^4}{192} - \frac{z^6}{2304} + \mathcal{O}(z^8 \log z)\right) \\ &= -\frac{1}{6\pi}\, x^2\left(1 - \frac{2}{x} + \frac{3}{x^2} - \frac{6}{x^3} + \mathcal{O}(x^{-4}\log x)\right) \end{aligned} \tag{10}$$

near $z = 0$. Thus, the general solution to (9) is $v(x) = \mu\hat{v}(x)$, with

$$\hat{v}(x) = -\frac{2}{3}\left(1 - \frac{2}{x} + \frac{3}{x^2} + \frac{3}{x^3} + \frac{C_1}{x^2}Y_4(i\sqrt{24/x})\right) + \frac{C_2}{x^2}J_4(i\sqrt{24/x}). \tag{11}$$

While v is regular at $x = \infty$ for any values of the constants C_1, C_2, equation (7) shows that in order for $\dot{k}$ to be regular at $x = \infty$, it is necessary that v decays at least as fast as x^{-2}. Comparing the expansion (10) with (11) one sees that by choosing $C_1 = 6\pi$, one can get rid of all terms which decay slower than x^{-3}. By looking at gauge-invariant expressions, we will show later that this choice is indeed necessary in order to get an asymptotic flat solution. Therefore, C_1 is fixed by physical means. The role of the constant C_2 is discussed below.

C. The Zerilli amplitudes

In Ref. [6], we have recently derived a gauge-invariant generalization of the Zerilli equation which allows to study perturbations on a Schwarzschild background written in any spherically symmetric coordinates. In Appendix A of this paper we have written the perturbed metric in terms of the generalized Zerilli function ψ, that one obtains using the formalism of [6] for the case at hand: $l = 2$, even parity perturbations of a KS background. This metric is a solution of Einstein's vacuum equations provided the Zerilli function ψ satisfies

$$\ddot{\psi} = \frac{x-2}{x+2}\psi'' + \frac{2}{x(x+2)}(\psi' - \dot{\psi}) + \frac{4}{x+2}\dot{\psi}' - \frac{6(3+6x+4x^2+4x^3)}{x^2(x+2)(3+2x)^2}\psi \tag{12}$$

where $\psi = \psi(\tau, x)$, $\tau := t/M$, and now $\dot{\psi} = \partial_\tau \psi$, $\psi' = \partial_x \psi$.

In order to evolve the KS initial data, we have to relate the amplitudes v, $\dot{v}$ and $\dot{k}$ to the scalars ψ and $\dot{\psi}$ (introduced in [6]) which satisfy the Zerilli equation (12). Using the expansions (4,5) in the expressions (1,2), it is straightforward to calculate ψ and $\dot{\psi}$ using the formulae given in [6]. The result is

$$\begin{aligned} \psi &= M\mu \left[\frac{x^3 \hat{v} - 6}{3x(2x+3)} \right], \\ \dot{\psi} &= -\mu \left[\frac{2x^3(2x-1)\hat{v} + x^4(x-2)\hat{v}' + 6(x-1)}{6x^2(2x+3)} \right], \end{aligned} \tag{13}$$

where we have also used (7) in order to eliminate $\dot{k}$. Since ψ is gauge-invariant, it is clear that the free constants C_1 and C_2 appearing in $v(x)$ cannot represent a gauge freedom. In order to have an asymptotic flat solution, $v(x)$ must vanish at infinity. As discussed in the previous subsection, this fixes the value of the constant C_1. In this case, ψ and $\dot{\psi}$ fall off like x^{-2} at infinity. The constant C_2 is still free and will determine different sets of initial data as one chooses its value. The radiation content, as we will see, depends on C_2. The constant is therefore clearly associated with the "spurious radiation" that the initial data contains with respect to "astrophysically relevant" initial data. One could probably determine this content by evolving the initial data set backwards in time. This calculation would be possible (at least for a limited amount of time) within the confines of the close approximation if the black holes are initially very close. One could therefore follow the space-time backwards for a short time and see if incoming radiation is present at a finite distance of the holes. We have not performed such a study, but it is feasible (we thank Jeff Winicour for bringing this to our attention).

It should be noticed that the initial data for the Zerilli function diverges in the limit $x \to 0$ for all values of C_2, so one cannot single out a preferred value of this constant by demanding the initial data to be finite in this limit (even though, as already mentioned, the multipole expansion is, in any case, valid only for $r > \max\{a_1, -a_2\}$).

D. The linearized apparent horizon equation

Bishop *et al.* have argued that the position of the apparent horizon is related to the constant C_2. Here we perform a linearized analysis of the position of the horizon, to clarify the meaning of their finding.

Given initial data $\bar{g}_{ij}$, K_{ij} on a space-like slice Σ, the location of an apparent horizon (AH) can be determined by the equation

$$\bar{\nabla}_i s^i - K_{ij} s^i s^j + K = 0, \tag{14}$$

where s^i is the unit outward normal to the AH. If the AH is given by $f(\underline{x}) = 0$ for some function f on Σ, we have $s_i = \lambda \bar{\nabla}_i f$, where $\lambda^{-2} = \bar{\nabla}^i f \bar{\nabla}_i f$. For KS initial data of the form (1,2), it was assumed in [5] that the AH coincides with a surface S which is orthogonal to k_i. In this case, $s_i = -k_i/\alpha$, and one can check that equation (14) is compatible with the result in [5], i.e. S is an AH if $V = -1/2$ on S.

For spherically symmetric initial data,

$$\begin{aligned} \bar{g}_{ij} dx^i dx^j &= \gamma(r)^2 dr^2 + r^2 d\Omega^2, \\ K_{ij} dx^i dx^j &= p(r)\gamma(r)^2 dr^2 + q(r) r^2 d\Omega^2, \end{aligned}$$

we must have $s_r = \gamma$, $s_A = 0$, and the AH equation (14) yields $q = -1/(r\gamma)$. It is not difficult to show (either by using the KS form of the Schwarzschild metric or more generally by integrating the constraint equations) that this is equivalent to $r = 2M$, where M is the ADM mass. We now want to linearize the AH equation around a spherically symmetric background and to find the deviation of the AH from $r = 2M$. In the linear regime, we expect that the location of the AH can be described by the image of the circle $|\underline{x}| = 2M$ under a map of the form

$$\underline{x} \mapsto \underline{x} - \epsilon^2 D(\underline{x}) \frac{\underline{x}}{r}.$$

The deviation function $D(\underline{x})$ is related to the function $f(\underline{x})$ as follows:

$$0 = f\left(\underline{x} - \epsilon^2 D(\underline{x})\frac{x}{r}\right) = f^{(0)}(\underline{x}) + \epsilon^2\left[\delta f(\underline{x}) - \partial_r f^{(0)}(\underline{x}) D(\underline{x})\right],$$

where $f^{(0)}(\underline{x})$ is a function describing the AH to zeroth order (for example $f^{(0)}(\underline{x}) = r - 2M$). Using also $\lambda = \gamma(\partial_r f)^{-1}$, we obtain $D(\underline{x}) = \lambda\delta f/\gamma$.

On the other hand, for linear perturbations around a spherically symmetric background, the normalization of s^i yields $\delta s_r = \gamma h/2$ while equation (14) gives

$$0 = \hat{\nabla}^A\left(\gamma\delta s_A - q_A\right) - rh + \frac{r^2}{2}k' + \gamma r^2 V_k\,, \tag{15}$$

where h, k, q_A and V_k are defined by

$$h = \gamma^{-2}\delta g_{rr}\,,\quad q_B = \delta g_{rB}\,,\quad k = \bar{g}^{AB}\delta g_{AB}\,,\quad V_k = \delta(\bar{g}^{AB}K_{AB}).$$

In terms of the function f, $s_A = \delta(\lambda\partial_A f) = \gamma\partial_A D$, and the linearized AH equation finally becomes

$$\gamma^2\hat{\Delta}D = \hat{\nabla}^A q_A + rh - \frac{r^2}{2}k' - \gamma r^2 V_k\,. \tag{16}$$

Performing a multipolar decomposition of $D(\underline{x})$, this equation becomes a set of algebraic equations for D. Evaluating for the KS data proposed in Section II A, we find that

$$D(\underline{x}) = \alpha^2\left(\frac{\alpha^2}{3}(3r + 2M)v(r) - \frac{4MM_1M_2}{r^2}\right)P_2(\cos\vartheta). \tag{17}$$

So we see that the position of the apparent horizon is given by the image of the circle $x = 2$ under the map

$$\underline{x} \mapsto \underline{x} - \varepsilon^2\hat{D}(\underline{x})\,\frac{x}{x}$$

where the deviation function $\hat{D}(\underline{x})$ can be expressed algebraically in terms of the perturbed three metric and extrinsic curvature. For a KS metric and $x = 2$,

$$\hat{D}(\underline{x}) = \frac{\mu}{2}\left(\frac{4}{3}\hat{v}(2) - 1\right)P_2(\cos\vartheta).$$

It is now clear that the deviation function depends on the value of the constant C_2. In particular, we can choose C_2 such that $\hat{D}(x = 2)$ vanishes.

A way to see that the meaning of the constant C_2 is not just a choice in the position of the apparent horizon is to notice that once one has fixed the values of M_1 and M_2, the KS form of the metric completely fixes the coordinates on the KS slice (at least to linear order). Indeed, a gauge mode must satisfy the constraint equations (6,7,8). On the other hand, the general solution to these equations is completely determined by $v(x)$, which cannot contain gauge modes since it is related to the gauge-invariant amplitude ψ according to (13). Therefore, we have two possibilities to fix the coordinate location of the apparent horizon (at $x = 2$, say): The first possibility is to perform an infinitesimal coordinate transformation such that the apparent horizon appears unperturbed relative to the Schwarzschild horizon. In this case, the initial data will not have KS form anymore. The second possibility is to adjust the constant C_2 such that the apparent horizon is at the location we desire. Clearly, these two methods are different since the former corresponds to a gauge transformation, while the latter corresponds to a true physical change in the initial data, as we anticipated before. So indeed the constant C_2 is related to the position of the apparent horizon as was noticed by Bishop *et al.*, but through a genuine change (not just a gauge change) in the initial data.

Figures 1 and 2 show the initial data for the Zerilli function and its time derivative for different values of C_2. In the next section we shall analyze the dependence of the total radiated energy and waveforms on C_2.

III. EVOLUTION

An expression for the radiated energy in terms of gauge-invariant quantities is given in [6]. Since here we have expanded all perturbations with respect to the Legendre polynomial $P_2(\cos\vartheta) = \sqrt{4\pi/5}\,Y^{20}(\vartheta)$, this energy expression becomes

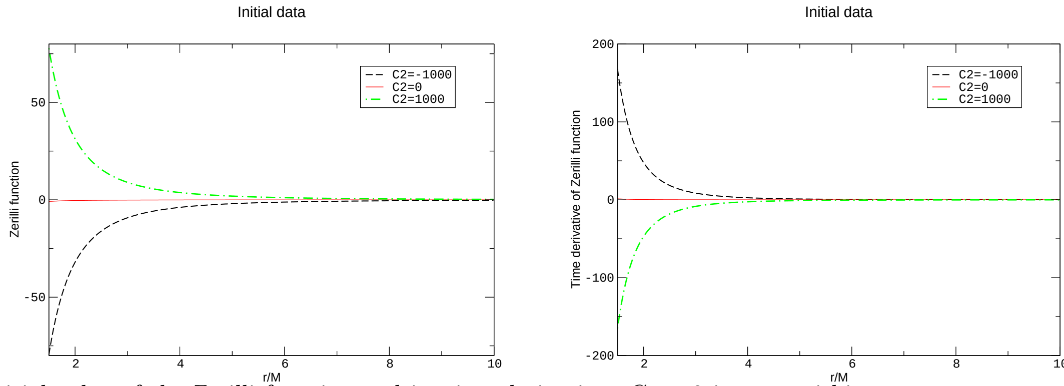

FIG. 1. The initial value of the Zerilli function and its time derivative. $C_2 = 0$ is nonvanishing, as may appear due to the choice of scale of the figures.

$$\frac{dE}{du} = \frac{6}{5}\dot{\psi}^2 ,$$

with $\dot{\psi}$ being evaluated in the radiative zone.

We have written a code that solves our generalized Regge-Wheeler and Zerilli equations. As a consistency check, we have evolved the close limit of some maximally sliced initial data (which can be seen as perturbations of Schwarzschild in usual coordinates), being able to reproduce previous values for the total radiated energy (e.g. Misner's initial data [8], or boosted black holes [9]).

The code is a standard second order dissipative, finite differencing, one. In the case of a KS background we perform excision, i.e. we place the inner boundary inside the black hole, and in that way avoid giving boundary conditions there. In figure 3 we show the Zerilli function, scaled by μ (i.e. ψ/μ) versus time, extracted at $r = 100M$ for two different values of C_2. From that plot one can notice, for example, the typical ringing frequency for Schwarzschild black holes.

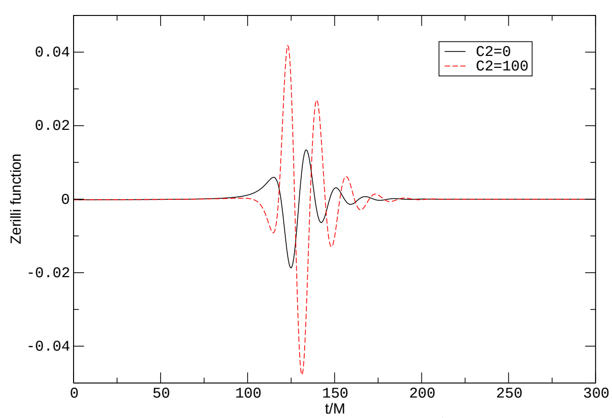

FIG. 2. The radiated waveforms at $r = 100M$ for two different values of C_2. As usual in close limit collisions, the waveform is dominated by the fundamental quasi-normal mode.

Given the linearity of Zerilli's equation, and the form of the KS initial data, the dependence of the Zerilli function on the parameters of the problem is

$$\dot{\psi}(t,r) = \epsilon^2\mu\left(\dot{\psi}_a(t,r) + C_2\dot{\psi}_b(t,r)\right)$$

where the functions ψ_a and ψ_b are dimensionless. Accordingly, the total radiated energy is

$$E = \frac{6\epsilon^4\mu^2}{5}\left(\int_0^\infty \dot{\psi_a}^2 dt + C_2^2\int_0^\infty \dot{\psi_b}^2 dt + 2C_2\int_0^\infty \dot{\psi}_a\dot{\psi}_b dt\right)$$

Therefore one needs to perform only three runs to obtain the complete dependence of the radiated energy on the free

parameters. The result is

$$E = \epsilon^4 M\mu^2 \left(3.6 \times 10^{-4} + 5.2 \times 10^{-7} C_2^2 - 2.2 \times 10^{-5} C_2\right)$$

The quantity $E/(\epsilon^4 M\mu^2)$, as a (quadratic) function of C_2, has a local minimum at $C_2 \approx 21$, where it takes the value $E/(\epsilon^4 M\mu^2) \approx 1.3 \times 10^{-4}$. This function is plotted in figure 3.

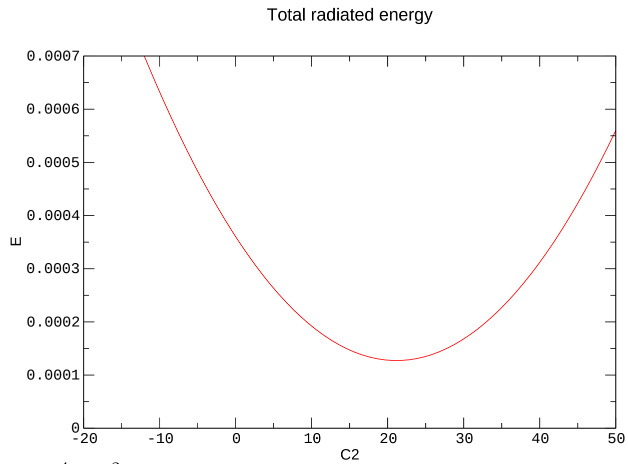

FIG. 3. The radiated energy (in units of $\epsilon^4 M\mu^2$) as a function of C_2. As a rough comparison, if we consider equal mass black holes ($\mu = 1$), and take the "separation" $\epsilon = 1$ and identify it with the "separation in the conformal background geometry" for two Brill–Lindquist black holes (for a more physical picture, a conformal separation of less than $0.8M$ corresponds to a common apparent horizon for the Brill–Lindquist family), the latter would radiate around $10^{-5}M$, which is rougly similar to the radiation we get for the minimum value of C_2.

IV. DISCUSSION

We have evolved the initial value family of Bishop *et al.* in the limit in which the black holes are close to each other by treating the spacetime as a single distorted Kerr–Schild black hole and solving the linearized Einstein equations for the distortion. The evolution sheds further light on the role of the integration constants present in the family.

An obvious question to ask would be "does this family contain more/less radiation than other families" (for instance the Misner data). Unfortunately, the family has explicit free parameters and therefore the comparison is highly dependent on the arbitrary values of these parameters. This should not be misinterpreted as a problem: it just highlights that the initial value problem for binary black holes inevitably contains ambiguities. Some proposals may resolve the ambiguities based on aesthetic criteria, but from a physical point of view that is not more satisfactory than simply picking values for the constants involved. These issues could be better understood if one evolved the systems backwards in time and tried to establish the amount of incoming radiation. The present results should be of interest in the calibration of numerical codes based on the Kerr-Schild coordinate system. Experiments with the Maya binary black hole code [10] to compare results are currently under way.

ACKNOWLEDGMENTS

This work was supported in part by the Swiss National Science Foundation, by grants NSF-PHY-0090091, NSF-PHY-9800973, by Fundación Antorchas, by the Eberly Family Research Fund at Penn State and the Horace C. Hearne Jr. Institute of Theoretical Physics. We wish to thank Jeff Winicour for comments and for helping us clarify an earlier version of the paper. After completing this work we learnt that Carsten Köllein at Albert Einstein Institute performed a perturbative evolution of this same family of initial data in his M.Sc. thesis (unpublished) using the Teukolsky equation. We thank Manuela Campanelli for bringing this to our attention.

APPENDIX A: PERTURBED METRIC FOR A KS BACKGROUND

For the particular case in which the background metric is KS, the perturbed $l = 2$ even metric in the Regge–Wheeler gauge, in terms of the Zerilli function, is

$$g_{rr} = 1 + \frac{2}{x} + \frac{\delta}{M}\left[\frac{6(x+2)(3+6x+4x^2+4x^3)}{(3+2x)^2x^3}\psi + \frac{4(4x^2+9x+3)}{x^2(3+2x)}\dot{\psi} - \right.$$
$$\left. \frac{2(2x^3+2x^2+15x+6)}{x^2(3+2x)}\psi' - 2x\psi''\right] Y^{20}(\vartheta)$$

$$g_{rt} = \frac{2}{x} + \frac{\delta}{M}\left[\frac{12(3+6x+4x^2+4x^3)}{x^3(3+2x)^2}\psi + \frac{4(2x^2-6x-3)}{x^2(3+2x)}\psi' - \right.$$
$$\left. \frac{2(x+2)(2x^2-3-6x)}{x^2(3+2x)}\dot{\psi} - 2x\dot{\psi}'\right] Y^{20}(\vartheta)$$

$$g_{\theta\theta} = x^2 + \frac{\delta}{M}\left[-\frac{12(x+1+x^2)}{3+2x}\psi - 4x\dot{\psi} - 2x(x-2)\psi'\right] Y^{20}(\vartheta)$$

$$g_{\phi\phi} = g_{\theta\theta}\sin^2\theta$$

$$g_{tt} = -1 + \frac{2}{x} + \frac{\delta}{M}\left[\frac{6(4+x^2)(3+6x+4x^2+4x^3)}{(3+2x)^2x^3(x+2)}\psi + \frac{4(5x^2+15x+6)}{x^2(3+2x)(x+2)}\dot{\psi} - \right.$$
$$\left. \frac{2(2x^4-2x^3-5x^2+24x+12)}{x^2(3+2x)(x+2)}\psi' - 2x\frac{x-2}{x+2}\psi'' - \frac{8x}{x+2}\dot{\psi}'\right] Y^{20}(\vartheta)$$

where δ is a perturbative parameter.

---

[1] G. Cook, Living Rev. Rel. **3**, 5 (2000).
[2] A. Garat and R. H. Price, Phys. Rev. D **61**, 124011 (2000).
[3] For an alternative approach that incorporates Kerr black holes with control on spurious radiation, plus the additional advantage of having proofs for the existence and regularity of the initial data, as well as knowing how to give boundary conditions to these data, see S. Dain and H. Friedrich in, gr-qc/0103030; gr-qc/0012023; gr-qc/0102047.
[4] R. A. Matzner, M. F. Huq and D. Shoemaker, Phys. Rev. D **59**, 024015 (1999).
[5] N.T. Bishop, R. Isaacson, M. Maharaj, and J. Winicour, Phys. Rev. **D57**, 6113 (1998).
[6] O. Sarbach and M. Tiglio, Phys. Rev. **D64**, 084016 (2001).
[7] W. Walter, *Ordinary Differential Equations* (Springer, New York 1998).
[8] R. Price and J. Pullin, Phys. Rev. Lett. **72**, 3297 (1994).
[9] J. Baker et. al. Phys. Rev. D **55**, 829 (1997).
[10] `http://www.astro.psu.edu/nr/`